TITLE: DYNAMIC VEHICLE DISPATCHING AT INTERMODAL TRANSFER STATION

by

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ABSTRACT

Transfer time is one of the most important service quality indicators for evaluating intermodal transit system. In the advent of Advanced Public Transportation System (APTS) (e.g. AVL and ATIS), vehicle arrival times and transfer demand can be obtained in real time. Thus, the dispatching decision of vehicles at transfer station can be made in a dynamic way to reduce transfer cost.

The purpose of this study is to improve transit service quality by reducing transfer time that can be achieved through dynamic dispatching of vehicles on connecting routes at transfer station. The time varying total cost function, including the connection delay and missed connection cost incurred by transfer passengers, and vehicle holding cost is formulated as a function of holding times for vehicles that are ready to be dispatched at transfer stations. A procedure is developed to dynamically optimize the dispatching time for each ready vehicle by minimizing the time varying objective function. A numerical example consisting of four routes connecting at a transfer terminal is designed to demonstrate the application of the dispatching model. It is found that the proposed method can be used to significantly reduce the transfer time.

Keywords: Transit, Vehicle Dispatching, Transfer, Optimization
1. INTRODUCTION

Transfer time is one of the most important service quality indicators for evaluating intermodal transit system. Efficient transfer connection among vehicles may significantly improve the service quality, stimulate demand and increase productivity. Due to stochastic headway variations, pure schedule synchronization of vehicles among connecting routes at transfer station cannot significantly reduce transfer time (Chowdhury 2000). In the advent of Advanced Public Transportation System (APTS) (e.g. AVL and ATIS), it is possible to predict the vehicle arrival times and demand in real time (Federal Transit Administration, Update ‘98). Thus, the dispatching of vehicles at transfer station can be determined in a dynamic way to improve the transfer efficiency.

The purpose of this study is to improve transit service quality by reducing transfer time that can be achieved through dynamic dispatching of vehicles of various connecting routes at a transfer station (See Figure 1). The dynamic total cost function, including the connection delay and missed connection costs incurred by transfer passengers, and vehicle holding cost, is formulated and discussed in this study. The optimal vehicle holding time is determined by minimizing the total cost function. In addition, a procedure is developed for dynamically evaluating vehicle dispatching times.

2. LITERATURE REVIEW

Many previous studies addressed the applications of control to the operation of transit vehicles for improving service reliability. Abkowitz and Tozzi [1997] examined the impact of ridership profiles on the effectiveness of headway control. Five boarding and alighting profiles were examined: (1) boarding at the beginning of the route and alighting at the end of the route; (2) boarding at the beginning of the route and alighting at the middle and end of the route; (3) boarding at the beginning of the route and alighting at the middle of the route; (4) boarding and alighting uniformly along the route; and (5) boarding at the middle of the route and alighting at the end of the route. They found that the implementation of headway control achieved the greatest reduction of wait time when passengers were boarding at the middle of the route and alighting from vehicles at the end of the route.

Abkowitz, Josef, and Driscoll [1987] developed a computer simulation model programmed by FORTRAN to evaluate four timed transfer strategies: (1) unscheduled transfers, (2) scheduled transfers (without vehicle holding), (3) scheduled transfers (holding vehicles operating on a low frequency route until vehicle operating on a higher frequency route arrive), and (4) scheduled transfers (always holding the early arriving vehicle). They simulated a network which consisted of two routes with a single transfer point and found that route characteristics including scheduled headway, percentage of transferring and through passengers at the transfer point and distance from the route origin to the transfer point play significant roles in determining a preferable transfer strategy. Simulation results showed that
double holding strategy was preferable when headway of both routes were equal, while scheduled strategy was preferable when headways of both routes are very high.

Dessouky, Hall, Nowroozi, and Mouriks [1999] developed a simulation model for assessing various bus holding strategies at timed transfer stations. Several holding strategies were examined: (1) holding a vehicle until all coordinated vehicles arrive; (2) dispatching the vehicle at its schedule departure time; (3) holding the vehicle until predefined fixed period; and (4) holding the vehicle until predefined fixed period if at least one late vehicle is predicted to arrive during the holding period. Simulation results showed that real time vehicle arrival information significantly reduce vehicle departure lateness without increasing number of passengers missing their connections.

Lee and Schonfeld [1994] formulated a model for holding or dispatching decision of coordinated vehicles at a transfer terminal. Holding times were optimized by minimizing the total cost, which includes operator cost of holding vehicle, holding cost of onboard passengers and missed connection cost of passengers from late connecting vehicles. In that model, missed connecting passengers wait cost was assumed to be equal to scheduled headway and connection delay cost of passengers from late incoming vehicles were neglected.

Intelligent Transportation Systems (ITS) technologies can significantly improve passenger intermodal operations and services [Miller and Tsao, 2000]. The computer-aided bus dispatching system has been in practice or recently implemented in many transit industries [Federal Transit Administration, Update ‘98]. Tri-Met [Strathman, et. al, 1999], the transit provider in Portland, has implemented satellite-based Global Positioning Systems (GPS) to track vehicle locations. Bus dispatchers has used real time bus locations and scheduled deviation information in order to dispatch buses. Ann Arbor (Michigan) Transportation Authority (AATA) deployed advanced public transportation system (APTS) technologies in its bus transit routes [Levine, Hong, Gug, and Rodriguez 2000]. The system called “Advanced Operating System” (AOS) enabled digital bus-to-bus communications to improve the transfer between buses. Buses among coordinated routes can locate other vehicle positions through the digital communication system and can request for holding earlier arrived vehicles to ensure successful connection of passengers from the late vehicles. The system did not optimize holding time; rather a preset maximum (up to five minutes) holding is executed. It was reported that the system is capable of improving transfer efficiency. The literature review gives an overview of the past efforts on vehicle holding strategies in transit services and signifies that there is a need for the development of dynamic dispatching model in order to gain maximum benefits from APTS technologies and to improve the transfer efficiency in the intermodal transit services.

3. METHODOLOGY

The objective total cost function is defined by the sum of vehicle holding cost, connection delay cost, and missed connection cost. The dispatching decision for each vehicle
is evaluated at the time before dispatching the vehicle. If the vehicle is held for connecting a late vehicle, its dispatching time will be re-evaluated periodically (e.g., every 30 seconds in this study) until the holding vehicle is dispatched. While evaluating the dispatching decision, the cost associated with holding the vehicle is independent on the dispatching decision of other vehicles those have arrived at the transfer station. Thus, the cost functions for dispatching vehicles can be determined individually. Hence, the optimal dispatching time (either with or without holding) of each vehicle can be determined by minimizing total cost.

In this study, two major assumptions are made for developing the cost functions. First, it is assumed that vehicle arrival times including the locations of late vehicles and their arrival distributions at transfer stations are available. In real world, vehicle arrivals at a transfer station are stochastic and deviate from their expected arrival times due to stochastic traffic condition and ridership at stations/ stops along the service route. Thus, vehicle arrival distributions at transfer stations will be highly location and time dependent. Normal and lognormal vehicle arrival distributions were observed in previous studies. The second assumption is assumed that the transfer demand from one vehicle to another is known or predictable. For the purpose long-term demand estimation, historic ridership information is required for developing a prediction model. However, for short term estimation, a new technology – automatic passenger counter systems can be applied provide time varying data.

To estimate late vehicles arrival time, a number of checkpoints (can be located at any bus stops) along the route can be designated. The travel time variation from any checkpoint to the transfer station can be accurately estimated by advanced arrival time prediction models: artificial neural networks (ANNs) developed by Chien and Ding (1999), multivariate regression models developed by Abdelfattah et. al. (1997) and Zeng et. al. (1999), and Kalman filtering models developed by Wall and Dailey (1999). To remember the arrival and departure time of each vehicle as well as the transfer demand information, a dynamic database will be accessed and updated with real time information such as transfer demand from one vehicle to another, vehicle arrival and departure times, and locations of late incoming vehicles.

Procedure for Dynamic Vehicle Dispatching

The dynamic dispatching procedure for each vehicle is activated at the time that a vehicle arrives at the transfer station. The step by step procedure for dynamic dispatching of vehicles is discussed below. Figure 4 shows the flow chart of dynamic dispatching procedure.

Step 1: At the time before dispatching vehicle v at the transfer station, estimate transfer demand from late vehicles to the vehicle.

Step 2: Estimate the means and the standard deviations of late vehicle arrival times to the transfer station according to their locations on their service routes. If all late vehicles have arrived, dispatch vehicle v immediately. Otherwise, go to Step 3.

Step 3: Optimize the holding time ($t^h_v$) of vehicle v by minimizing the objective
total cost function \((TC_v)\) including holding delay, connection delay and missed connection costs that will be discussed in the next section.

Step 4: If optimal holding time is less than or equal to evaluation interval \((\Delta)\), dispatch vehicle \(v\) at the end of optimal holding time. Otherwise, go to Step 1.

The development of the total cost function applied in this procedure will be introduced next, while the associate cost components considered in this study will be formulated and discussed.

**Model Formulation**

Based on definition, the objective total cost function for dynamic vehicle dispatching is the sum of vehicle holding cost, connection delay cost, and missed connection cost. Variables used to formulate the total cost function are defined in Table 1. The total cost \(TC_v\) for dispatching vehicle \(v\) at a transfer station is formulated as

\[
TC_v = C_v^O + C_{b,v}^C + C_{b,v}^M
\]  

(1)

where \(C_v^O\), \(C_{b,v}^C\), and \(C_{b,v}^M\) represent the vehicle holding cost, the connection delay and missed connection costs caused by holding vehicle \(v\) for waiting a late vehicle \(b\), respectively.

**Vehicle Holding Cost**

In order to evaluate the holding decision for holding a ready vehicle, the vehicle holding cost is the holding time multiplied by the vehicle operating cost as formulated in Eq. 2.

\[
C_v^O = t_v^h u_b
\]  

(2)

where \(v\) represents the holding vehicle, while \(t_v^h\) and \(u_b\) are holding time and vehicle operating cost, respectively.

**Connection Delay Cost**

The connection delay cost is incurred by transfer passengers arriving between the departure times of vehicles \(v-1\) and \(v\), while vehicle \(v-1\) is the vehicle arriving at the transfer station prior to vehicle \(v\).

The connection delay cost \(C_{b,v}^C\) incurred by passengers transferring from the late vehicle \(b\) to the ready vehicle \(v\) is affected by the arrival distribution of vehicle \(b\) at the transfer station. \(C_{b,v}^C\) will be evaluated at dispatching decision point of time, \(t_{v}^{dd}\). For instance, if vehicle \(b\) arrives before \(t_{v}^{dd}\), the connection delay cost is simply the vehicle holding time \(t_v^h\),
multiplied by transfer demand $U_{b,v}$ from vehicle $b$ to $v$ and the value of users’ wait time. However, by taking into consideration of stochastic vehicle arrivals, if vehicle $b$ arrives after $t_{dd}^v$, the connection delay cost is formulated based on the probability that vehicle $b$ arrives before dispatching vehicle $v$. Therefore, the connection delay cost of transfer passengers from vehicle $b$ to vehicle $v$ is the transfer demand $U_{b,v}$ multiplied by the probability of vehicle $b$ will arrive between $t_{dd}^v$ and $t_{v}^d$ (as shown in Figure 2, area A), the corresponding wait time, and the value of users’ wait time $u_w$. Thus, the connection delay cost can be formulated as

$$
C_{b,v}^C = \begin{cases} 
    t_b^h \sum_b U_{b,v}u_w & \text{if } t_{v-1}^a < t_b^d \leq t_{dd}^v \\
    \sum_b \int_{t_b^d}^{t_b^d+t_b^h} f(t_b^a)[(t_{dd}^v + t_b^h) - t_b^a]dt_b^a U_{b,v}u_w & \text{if } t_{dd}^v < t_b^a \leq t_{dd}^v + t_b^h
\end{cases} \forall b \quad (3)
$$

where $f(t_b^a)$ and $U_{b,v}$ represent the probability distribution for the arrival of vehicle $b$, and the transfer demand from vehicle $b$ to $v$, respectively. In Eq. 3, the dispatching decision time (or re-evaluation time) can be determined after knowing the arrival time $t_v^a$ of vehicle $v$, the number of evaluation $n$ for holding vehicle $v$ and the evaluation interval $\Delta$ (e.g., 30 seconds), and formulated as

$$
t_{dd}^v = t_v^a + (n-1)\Delta \quad (4)
$$

For example, at the first dispatching decision time, $n$ is equal to 1. Thus, the first dispatching decision time of vehicle $v$ is $t_v^a$. The duration of $\Delta$ can be adjusted depending on the traffic condition over time.

**Missed Connection Cost**

The missed connection cost is incurred by the passengers arriving between $t_{dd}^v$ and $t_{v+1}^a$, and will miss vehicle $v$. The missed connection cost is formulated based on the probability of missed connection, which can be determined after knowing the vehicle arrival distribution. As shown in Figure 2, a missed connection occurs when the late vehicle $b$ arrives between $t_v^d$ and $t_{v+1}^a$ (area B). Thus, the missed connection cost $C_{b,v}^M$ is the transfer demand $U_{b,v}$ multiplied by the probability of missed connection, the corresponding wait time, and the value of users’ wait time:

$$
C_{b,v}^M = \sum_b \int_{t_v^a+1}^{t_{v+1}^a} f(t_b^a)[t_{v+1}^a - t_b^a]dt_b^a U_{b,v}u_w \quad \forall b \quad (5)
$$
The costs of connection delay and missed connection incurred by passengers transferring from late vehicles are affected by the arrival distribution of the late vehicles. Therefore, connection delay and missed connection cost formulated in Eqs. 3 and 5 may need to be reformulated if, for example, the arrival distribution of the late vehicles is lognormal \cite{Hines and Montgomery, 1990} rather than normal. Figure 3(a) shows that if vehicle \( v \) is dispatched \((t_{v}^{d} = t_{v}^{dd} + t_{v}^{h})\) before the earliest arrival time \( t_{b}^{e} \) of late vehicle \( b \), the connection delay cost can be ignored (the transfer passengers from late vehicle \( b \) will not take vehicle \( v \)). However, if vehicle \( v \) is dispatched after the earliest arrival times of vehicle \( b \) as shown in Figure 3(b), both connection delay and missed connection costs exist. The connection delay and missed connection costs, considering late vehicles arrival distribution with finite earliest arrival time, are formulated in Eqs. 6 and 7, respectively.

\[
C_{b,v}^{C} = \sum_{b} t_{b,v}^{C} U_{b,v} u_{w} \quad \forall b \tag{6}
\]

\[
C_{b,v}^{M} = \sum_{b} t_{b,v}^{M} U_{b,v} u_{w} \quad \forall b \tag{7}
\]

where \( t_{b,v}^{C} \) and \( t_{b,v}^{M} \) represent connection delay and missed connection delay times, and are formulated in Eqs. 8 and 9, respectively.

\[
t_{b,v}^{C} = \begin{cases} 
\int_{t_{b}^{e}}^{t_{v}^{dd} + t_{v}^{h}} f(t_{b}^{e}) [t_{v}^{dd} + t_{v}^{h} - (t_{b}^{e} + t_{v}^{a})]dt_{b}^{a} & \text{if } t_{v}^{dd} + t_{v}^{h} > t_{b}^{e} \quad \forall b \\
0 & \text{otherwise} 
\end{cases} \tag{8}
\]

\[
t_{b,v}^{M} = \begin{cases} 
\int_{t_{b}^{e}}^{t_{v}^{dd} + t_{v}^{h}} f(t_{b}^{e}) [t_{v}^{dd} + t_{v}^{h} - (t_{b}^{e} + t_{v}^{a})]dt_{b}^{a} & \text{if } t_{v}^{dd} + t_{v}^{h} > t_{b}^{e} \quad \forall b \\
\int_{t_{v}^{e}}^{t_{v}^{dd} + t_{v}^{h}} f(t_{v}^{e}) [t_{v}^{dd} + t_{v}^{h} - (t_{b}^{e} + t_{v}^{a})]dt_{b}^{a} & \text{otherwise} 
\end{cases} \tag{9}
\]

4. MODEL EVALUATION

The major purpose of this section is to demonstrate the application of the dynamic vehicle dispatching model. The results are obtained from a computer program coded in FORTRAN. The studied transfer station, as shown in Figure 1, is a transfer terminal for four routes. Vehicle arrivals from routes 1, 2 and 3 follow lognormal distributions, while vehicle arrivals from route 4 follow uniform distribution. The baseline values of design variables, such as the value of user’s wait time and vehicle operating cost, are given in Table 1, while the vehicle operational information (e.g. headways) and transfer demand from one vehicle to another are shown in Table 2.
In real world, the value of user’s time can be determined based on the average annual income of the residents in the service region. For estimating vehicle operating cost, the expenses of maintenance, administration, and insurance for operating a vehicle should be considered in addition to the labor and energy consumption costs.

In order to optimize dynamic vehicle dispatching time, assume that vehicle \( a \) on route \#1 arrives at the transfer station according to its schedule, while vehicles \( b \) and \( c \) on routes \#2 and \#3 are late (see Figure 1). Since vehicle arrival time from route 4 is deterministic, thus arrival times of vehicles from route 4 are always known. If the reference point of time is 00:00 (zero minute and zero second), the schedule time for the next arrival on route \#1 is 20:00 (i.e., 20 minutes). Given that vehicle arrival times from route 1 thorough 4 are synchronized and at the dispatching decision point of time of vehicle \( a \), vehicles \( b \) and \( c \) are 0.51 and 1.02 miles away from the transfer station, respectively, the holding time for vehicle \( a \) need to be optimized. The travel time information of vehicles \( b \) and \( c \) from their locations to the transfer station are shown in Table 3.

According to the given information (e.g., travel times and transfer demand), the holding time for dispatching vehicle \( a \) at first dispatching decision time (i.e. 00:00) can be optimized by minimizing the total cost function formulated in Section 4. The maximum holding time can be determined by considering that both vehicles \( b \) and \( c \) are successfully connected with vehicle \( a \). Based on an incremental line search procedure (set size = 2 seconds), the total costs for different holding times are calculated. The global minimum total cost and the corresponding holding time can be identified.

At the first dispatching decision time, the optimal holding time of vehicle \( a \) is found to be 5.08 minutes, which yield the global minimum total costs of $21.19. However, under no holding situation, the total cost is $29.88. Therefore, the departure time of vehicle \( a \) is 05:05, while the relationship between the total cost and holding time is shown in Figure 5. We found that the total cost with respect to holding time is a non-convex function. From Figure 5, three local minimum points at the intersect points of lines \( A \), \( C \) and \( E \) and the total cost curve can be identified, while point \( C \) is found to be the global minimum point. Therefore, the optimal holding time of vehicle \( a \) is thus determined. The relationship between various cost components including the connection delay cost incurred by passengers transferring from route 4, the connection delay and missed connection costs incurred by passengers transferring from vehicles \( b \) and \( c \), and the holding cost of vehicle \( a \), and holding time is shown in Figure 6.

Since vehicle arrival times may vary from time to time due to incidents (e.g. passenger demand, vehicle breakdown and roadway congestion), the dispatching decision should be re-evaluated during the holding period. Assuming that the dispatching decision will be re-evaluated at a 30-second interval, the second dispatching decision (re-evaluation) time will be 00:30. At the second dispatching decision time, it requires to update every late vehicle location, and re-estimating the late vehicle arrival times to the transfer station. In order to
examine the sensitivity of the optimal holding time, three situations are considered at the second dispatching decision time as shown in Table 4.

The first situation shows that vehicle $b$ moves toward the transfer station during the re-evaluation interval, while vehicle $c$ does not move due to unpredictable reasons. The second situation shows that vehicle $c$ moves toward the transfer station during the re-evaluation interval, but vehicle $b$ does not move. The third situation shows that neither vehicle $b$ nor $c$ moves toward the transfer station during the re-evaluation interval.

At the second dispatching decision time, the holding time of vehicle $a$ is re-optimized based on the three situations. The optimal solutions of holding and departure times of vehicle $a$ for the three situations are discussed below.

**Situation 1:**

In this situation, the optimal holding time is found to be around 3.44 minutes (Figure 7), which yield the minimum total costs of $17.87. Therefore, the new departure time of vehicle $a$ evaluated at 00:30 is updated from its original 05:05 to 03:56. The new departure time of vehicle $a$ is earlier than that was determined at the first dispatching decision time, because vehicle $b$ is expected to arrive earlier than that was expected at first dispatching decision time.

**Situation 2:**

The optimal holding times under situation 2 is found to be 7.98 minutes (see Figure 8), which yield the minimum total costs of $19.43. The new departure time of vehicle $a$ reevaluated at 00:30 is updated from its original 05:05 to 08:29. The new departure time of vehicle $a$ is later than that was determined at the first dispatching decision time due to late arrival of vehicle $b$ and earlier arrival of vehicle $c$.

**Situation 3:**

The optimal holding time under situation 3 is found to be 5.08 minutes (See Figure 9), which yield the minimum total costs of $20.64. Therefore, the new departure time of vehicle $a$ evaluated at 00:30 is updated from the original 05:05 to 05:35. The departure time of vehicle $a$ is 30 seconds later than that was determined at first dispatching decision time because vehicles $b$ and $c$ did not move during the past evaluation interval. The iterative evaluation of holding time will be continued until the holding vehicle is dispatched.

5. CONCLUSIONS

A model for dynamic dispatching of vehicles operating in an intermodal transfer station has been discussed in this study. A numerical example, consisting of four route
connected to a transfer station, is presented, while holding times are optimized based on the
given vehicle arrival distributions, the delay of vehicle arrivals, and transfer demand. Based
on the results obtained from the dynamic vehicle dispatching models developed in this study,
the following conclusions are made.

1. The optimal holding time is a trade off among several cost components including
connection, missed connection and operator costs.
2. Dynamic vehicle dispatching can significantly improve the transfer efficiency.
3. At each vehicle dispatching decision point of time, the optimal holding time of a
vehicle depends on the arrival time of other connecting vehicles and transfer demand.
In general, holding for a late vehicle is preferred if that vehicle carries a large enough
number of transfer passengers.
4. Vehicle holding is preferable when uncertainty in the arrivals of late vehicles is small.
As the standard deviation of vehicle arrival times increases, the holding cost also
increases.
5. Excessive fluctuation in predicted arrival times and transfer demand may reduce the
benefit from holding a vehicle, specially when the delay of vehicle arrival is large and
the standard deviation of vehicle arrival times is high.

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LIST OF TABLES

Table 1 Notation
Table 2 Transfer Demand and Headways
Table 3 Late Vehicles Travel Time at First Dispatching Decision Time
Table 4 Vehicle Travel Times at the Second Decision Time

LIST OF FIGURES

Figure 1 Transfer Station with Connecting Routes
Figure 2 Probability Distribution of Late Vehicle Arrivals
Figure 3 Late Vehicle Arrival Distribution (Lognormal)
Figure 4 Dynamic Vehicle Dispatching Procedure
Figure 5 Total Cost vs. Holding Time
Figure 6 Various Cost Components vs. Holding time
Figure 7 Total Cost vs. Holding Time (Situation 1)
Figure 8 Total Cost vs. Holding Time (Situation 2)
Figure 9 Total Cost vs. Holding Time (Situation 3)
Table 1 Notation

\[ C_v^O = \text{Cost for holding vehicle } v (\$); \]
\[ C_{b,v}^C = \text{Bus-to-bus connection delay cost for holding vehicle } v (\$); \]
\[ C_{b,v}^M = \text{Bus-to-bus missed connection cost for holding vehicle } v (\$); \]
\[ TC_v = \text{Total cost for dispatching vehicle } v \text{ on route } j \text{ at stop } s (\$); \]
\[ t_b^e = \text{Earliest arrival time of vehicle } b \text{ at station } i \text{ (hr)}; \]
\[ t_v^h = \text{Holding time of vehicle } v \text{ (hr)}; \]
\[ t_{v-1}^d = \text{Departure time of vehicle } v-1 \text{ (hr)}; \]
\[ t_v^d = \text{Departure time of vehicle } v \text{ (hr)}; \]
\[ t_v^{dd} = \text{Dispatching decision time for vehicle } v \text{ (hr)}; \]
\[ t_{v+1}^a = \text{Arrival time of vehicle } v+1 \text{ (hr)}; \]
\[ t_v^a = \text{Arrival time of vehicle } v \text{ (hr)}; \]
\[ t_{b,v}^c = \text{Connection delay time of passengers from late vehicle } b \text{ to } v \text{ (hr)}; \]
\[ t_{b,v}^M = \text{Missed connection delay time of passengers from late vehicle } b \text{ to } v \text{ (hr)}; \]
\[ U_{b,v} = \text{Transfer demand from vehicle } b \text{ to vehicle } v \text{ (pass)}; \]
\[ u_w = \text{Value of users’ wait time (7 \$/hr)}; \]
\[ u_b = \text{Average bus operating cost (70 \$/hr)}; \]
\[ \Delta = \text{Evaluation interval for dispatching vehicle (sec)}; \]

Table 2 Transfer Demand and Headways

<table>
<thead>
<tr>
<th>Form</th>
<th>To</th>
<th>Average Transfer Demand (vehicle to vehicle)</th>
<th>Headway (minutes)</th>
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<tr>
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<td>Route 1</td>
<td>-</td>
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### Table 3 Late Vehicles Travel Time at First Dispatching Decision Time

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<th>(^4\text{SD of Travel Time (min)})</th>
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<td>c</td>
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<td>6.92</td>
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\(^1\text{VL= Vehicles location (distance from Transfer station)}\)
\(^2\text{MTT = Mean Travel Time}\)
\(^3\text{STT = Shortest Travel Time}\)
\(^4\text{SD = Standard Deviation}\)

### Table 4 Vehicle Travel Times at the Second Decision Time

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\(^1\text{VL= vehicle location (distance to the transfer station)}\)
\(^2\text{Travel Time: from current location to the transfer station}\)
\(^3\text{STT = mean travel time}\)
\(^4\text{SD = shortest travel time}\)
\(^5\text{SD = standard deviation}\)
Figure 1 Transfer Station with Connecting Routes
Figure 2 Probability Distribution of Late Vehicle Arrivals

\[ f(t^a_b) \]: probability density function of bus b’s arrival time

\( t^{dd}_v \): dispatching decision time of vehicle v

\( t^h_v \): holding time of vehicle v

\( t^{a}_{v+1} \): arrival time of vehicle v+1
Figure 3 Late Vehicle Arrival Distribution (Lognormal)

(a) $t_{v+1}^a > t_b^c > t_v^{dd} + t_v^h$

(b) $t_v^{dd} + t_v^h > t_b^c > t_v^{dd}$

$f(t_v^a)$ = arrival distribution of vehicle b

$t_v^{dd}$ = dispatching decision time of vehicle v

$t_v^h$ = optimal holding time of vehicle v

$t_b^c$ = earliest arrival time of vehicle b

$t_{v+1}^a$ = schedule arrival time of vehicle v+1
Figure 4 Dynamic Vehicle Dispatching Procedure
**Figure 5 Total Cost vs. Holding Time**
Figure 6 Various Cost Components vs. Holding Time

SC=dispatching delay cost
R4=connection delay cost incurred by passengers from vehicle of route 4
C-1= connection delay cost incurred by passengers from vehicle b
C-2= connection delay cost incurred by passengers from vehicle c
M-1= missed connection cost incurred by passengers from vehicle b
M-2= missed connection cost incurred by passengers from vehicle c
Figure 7 Total Cost vs. Holding Time (Situation 1)
Figure 8 Total Cost vs. Holding Time (Situation 2)
Figure 9 Total Cost vs. Holding Time (Situation 3)