A Methodology for Evaluating of School Bus Routing:
A Case Study of Riverdale, New Jersey

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ABSTRACT

School bus routing is a version of the traveling salesman problem with time-window constraints and route balancing. This paper evaluates and compares the results of three techniques used in school bus routing: a time savings heuristic, the ROUTER computer program, and the sweep method. These school bus routing techniques are evaluated using the case study of Riverdale, New Jersey. The case study involves a municipality with one elementary school and requires all of the buses to depart from and return to the school. The routes and operating costs vary for each of the methodologies used. Through analyzing different routing methodologies, this paper will provide guidelines to school districts and bus operators to develop an efficient and cost-effective school bus route and schedule.

KEYWORDS: School Bus Routing, Vehicle Routing and Scheduling, Traveling Salesman Problem
INTRODUCTION

Presently there are several methods used to solve the vehicle routing problem. Many of these methods can be applied to the school bus routing problem. However, due to the heuristic nature of the methodologies used, the near optimal solutions may vary. In this paper three heuristic methods will be evaluated to route the school buses in Riverdale, New Jersey. These are: a traditional time saving heuristic algorithm (1), a computer model (ROUTER), and the sweep method. Each method’s results will be reviewed and evaluated.

This problem and the constraints (or requirements) imposed on the bus operation can be translated into a mathematical model. The objective function of the model minimizes the operating cost while meeting the service requirements (e.g., each route should not be longer than 30 minutes) and time windows (e.g., each bus has to come back to the school before the classes start). The operating cost is a function of the speed of the vehicle, the distance between stops, the number of stops, the passenger dwell time, and the bus size. As it will be shown in the next section, these problems are very difficult to solve to the optimum solution and various heuristic methods are developed to determine a good minimum cost solution. The school bus routing constraints and optimization functions were used to develop a series of mathematical formula that shaped the objective of the study. The objective of this study is to evaluate the three proposed school bus routing methods and determine the method that minimizes the operating cost, which includes reducing the number of vehicles needed to serve all the bus stops.

School bus routing is a version of the traveling salesman problem, commonly referred to the category of vehicle routing problems (VRP), either with or without time window constraints. In addition to the numerous studies that addressed the vehicle routing problem, various software methods have been developed that can be utilized to minimize the operating cost. Three factors make school bus routing unique: efficiency (the total cost to run a school bus), effectiveness (how well the demand for service is satisfied) and equity (fairness of the school bus for each student). School bus routing has two separate routing issues - assigning students to bus stops and routing the buses to the bus stops. This study will focus on routing the buses to the bus stops.

The case study discussed in this paper involves the Riverdale, New Jersey elementary school where there are 199 students bussed to the school in the morning. The students were clustered into groups associated with 24 bus stops. The buses are required to depart and return to the school within an allotted half-hour time period subject to bus size constraints. This ensures that all students remained on the bus for an equitable time period.

LITERATURE REVIEW

Unique to school bus routing is the multi-objective nature involving the minimization of costs while maintaining user equity for the riders. Bowerman, Hall and Calamai (2) noted two problems with the traditional problem formation for school bus routing. Firstly, the problem formulation can not be solved in a reasonable amount of time for the solution of vehicle routing problems. Secondly, to illustrate the complex issue involving the problem formulation wherein a problem involving three routes with 100 students at 20 bus stops would require 3400 variables and 6800 constraints. They developed a heuristic solution to the school bus routing problem by first breaking it up into three subproblems. The first subproblem involves determining the location of school bus stops. The second subproblem allocates students to bus stops. Lastly, the routing subproblem determines the routes that visit the bus stops.
Numerous studies dealt with evaluating the performance of various heuristic algorithms for vehicle routing and scheduling. Ballou (3) reviewed the "savings," "clustering," and "sweeping" vehicle routing methods. He found that the savings method could reach the solution with approximately 2 percent error to the real optimal solution, with the clustering and sweeping methods having average errors of 13 percent. The cluster method involves grouping the stops together into routes. When the cluster method was changed from determining routes by the stops’ proximity to each other and introduced the vehicle capacity constraint, the error level dropped to approximately 8 percent. The sweep method involves using a rotating line to group the stops and generates routes. When the sweeping method was used in both counter-clockwise and clockwise directions, the error level was only reduced by 1 percent. However, even with the changes to the clustering and sweeping method, the savings method remained the best vehicle routing and scheduling method of the three.

Heuristic methods have been developed to provide the optimization for vehicle routing and scheduling problems that have time window constraints. Koskosidis and Powell (4) developed an algorithm for medium to large-size problems where the optimal solutions are not practical due to the time required to find the solutions. They explained that the largest problem solved optimally would be limited to 4 vehicles and 14 stops. Additionally, the authors acknowledge that heuristic methods may deviate 4 to 5 percent from the optimality. Also, vehicle routing with time window constraints require additional computation requirements.

Another area unique to school bus routing is the fixed routing problem. The paper by Savelsbergh and Goetschalckx (5) found that fixed routes require reduced management costs and reduced administrative costs, individual and predictable service, increased performance of drivers, and increased efficiency at the stops. Obviously, one disadvantage with fixed routing is the lack of flexibility in routing.

In addition to the traditional heuristic methods, there are a series of computer programs that can aid one in vehicle routing. The program ROUTER (6) determines routes and schedules for multiple vehicles serving multiple stops. ROUTER minimizes the total distance traveled on all routes while indirectly minimizing the total number of vehicles needed to serve the stops.

**METHODOLOGY**

There are three methodologies that will be applied to the case study to evaluate the operating cost of the school bus routing problem. These methods are the time savings heuristic method, ROUTER and the sweep method. These methods will be used to minimize the operating cost as described in the mathematical formulation for the school bus routing problem. All of these methodologies will be using the same data sets which include the same network, the same bus stops, and the same numbers in the school bus clusters. Additionally, each methodology will be using the same set of parameters regarding operating cost in relation to bus size and dwell time.

The three methodologies will be using the optimization criteria unique to school bus routing, which are load balancing involving equity on number of students per bus and length balancing which is the equity of the distance each student travels on each bus. The constraints are on this problem are bus capacity, route length and total travel time. The optimal school bus routing plan will be determined using two distinct routing methods: a heuristic algorithm and a computer program. The optimal routes determined from each method will then be analyzed and compared.
**Mathematical Formulation**

The school bus routing problem can be presented conceptually as follows:

The **Objective Function**: Minimize the total operating cost subject to the following constraints:

1. The time students spend traveling on the bus must not exceed a given, externally specified, limit.
2. The number of students per bus must not exceed the number of seats available on the bus.
3. Each bus stop is allocated to only one bus.
4. Every route must have at least one stop.
5. Each stop is allocated to only one route.
6. The number of buses leaving the school must equal the number of buses returning to the school.

This model can be written as the following mathematical problem:

**Sets**

- $T$: bus type available, $t \in T$
- $S$: bus stops with $i, j \in S$
- $O(S)$: the origin and destination of bus. $0 \in O$, $O \subset S$
- $L$: bus route with $k \in L$

**Parameters**

- $T_{\text{max}} = \text{maximum time available for the bus to pick up students on a route}$
- $s_{ij} = \text{distance between node } i \text{ and } j \text{ (in miles)}$
- $t_{d} = \text{dwell time of the bus at a node (in hrs)}$
- $d_{i} = \text{demand (number of students) to be picked up at node } i$
- $\delta_{k,t} = 1 \text{ if bus route } k \text{ has bus type } t, 0 \text{ Otherwise}$
- $O_{t} = \text{operating cost for type } t \text{ bus (in } \$/\text{bus-hr)}$
- $\text{Cap}_{t} = \text{seat capacity for type } t \text{ bus (in seats/bus)}$
Variables

$X_{ij,k} = 1$ if nodes $i$ and $j$ are catered consequently by bus $k$.

If $\sum_j X_{ij,k} = 1$, then $i$ is catered by bus $k$ (or $i$ is in bus route $k$).

Equations

The operating cost of bus route $k$ can be computed as:

$$\left( \sum_t \delta_{k,t} O_t \right)$$

The seat capacity of bus route $k$ can be computed as:

$$\sum_t \delta_{k,t} \times Cap_t$$

$t_k = \text{time taken for bus } k \text{ to pick up students on } k^{th} \text{ bus route and drop students at node 0}$ is computed as

$$t_k = \sum_i \sum_j (X_{ij,k} t_{ij} / \sum_t \delta_{kt} V_t) + 2 \times \sum_i (\sum_j X_{ij,k}) d_i t_d$$

Minimize

$$Z = \sum_{k=1}^J t_k \left( \sum_t \delta_{k,t} O_t \right)$$

Subject to

$$t_k \leq T_{\text{max}} \quad (1)$$

$$\sum_i (\sum_j X_{ij,k}) \times d_i \leq \sum_t \delta_{k,t} \times Cap_t \quad (2)$$
\[
\begin{aligned}
\sum_{k \in L} \sum_{j \in S} X_{ij,k} &= 1 \quad \forall i \in S, i \neq 0 \\
\sum_{k \in L} \sum_{j \in S} X_{ji,k} &= 1
\end{aligned}
\] (3)

\[
\begin{aligned}
\sum_{k \in L} \sum_{j \in S} X_{0j,k} &\geq 1 \\
\sum_{k \in L} \sum_{j \in S} X_{j0,k} &\geq 1
\end{aligned} \quad \forall k \in L
\] (4)

\[
\begin{aligned}
\sum_{j \in S} X_{0j,k} &\leq 1 \\
\sum_{j \in S} X_{j0,k} &\leq 1
\end{aligned} \quad \forall k \in L
\] (5)

\[
\sum_{j \in S} X_{ij,k} = \sum_{j \in S} X_{ji,k} \quad \forall k \in L, i \in S
\] (6)

Optimal number of buses of each type denoted as \( bus_t \) can be calculated as follows:

\[ Bus_t = \sum_j \delta_{jt} \times X_{0j} \] (7)

If \( bus_t \leq \sum_j \delta_{jt} \), \( \forall t \). Then, the solution is optimal.

Otherwise, \( bus_t = \sum_j \delta_{jt} \), \( \exists t \). Add one bus of type \( t \) to the bus route \( L \).

Considering the area where the school buses are to be routed as set of \( n \) nodes, each node represents a bus stop. The algorithm consists of solving a linear integer program iteratively with the number of buses of each type updated at each run. First, the number of bus types must be initialized with the total number of buses adequate for the number of students. Secondly, the mathematical formulation for the routing problem is solved for the total number of buses by bus type. Finally, a review of all the buses of certain bus type \( t \) is conducted.

**Time Saving Heuristic**
Heuristics are defined as a set of rules that are being followed in solving complex problems. In this particular case, they would reach a number of buses and the routes that are hopefully of the least cost nature. Clark and Wright's (1) saving heuristic is as follows:

**Step 1: Distance Matrix**

Establish the distance matrix \( S_{ij} \). For \( n \) bus stops, the matrix will be of the dimension \([n+1] \times [n+1]\). Set node 0 to represent the school, nodes 1 to \( n \) represent \( n \) bus stops.

**Step 2: Time Matrix**

On the basis of the distance matrix, calculate the time matrix \( T_{ij} \). Elements of the time matrix represent the travel time \( T_{ij} \) between any two nodes \( i \) and \( j \). \( T_{ij} \) is computed as \( d_{ij}/v \) where \( d \) is the distance between the nodes and \( v \) is the average running speed of the bus.

**Step 3: Time Saving Matrix**

The time saved by combining any two points into one route is computed as \( ts_{ij} \), where \( ts_{ij} = T_{0i} + T_{j0} - T_{ij} \) i,j > 0, \( ts_{ij} \) are the elements of the time saving matrix \( TS_{ij} \).

**Step 4: Generating Routes**

Choose the maximum positive element \( ts_{ij} \) in the matrix \( TS_{ij} \), connect \( i \) and \( j \), then select node \( i \) and node \( j \) as growth nodes. If the maximum time saving related to node \( i \) is greater than the maximum time saving related to node \( j \), node \( i \) will grow; otherwise, node \( j \) will grow. Check if the bus capacity constraint is met, and if the bus can reach the school in time when the node is added. Remove the corresponding row and column of the end nodes. Repeat step 4 until every node is included in the routes.

**ROUTER Computer Program**

ROUTER is a software program that was developed to "determine the best routes and schedules for a fleet of privately controlled vehicles" (6). ROUTER minimizes the total distance traveled on all routes while also minimizing the total number of vehicles needed to serve all the stops. While the software was designed for freight-vehicle routing, adjustments were made to allow the program to be used for school bus routing. The program includes mixed vehicle types, maximum time or distance on routes, earliest time for vehicle departure, latest time for vehicle return, dwelling times, time-windows, route costs based on fixed and variable rates, and incremental stop costs.

The location of each bus stop was designated by its coordinates, which are then used to compute the approximate straight-line distances. A circuitry factor is then applied to adjust the straight-line miles to approximate road miles. The default value is 1.21. Each bus stop was also assigned a dwelling time based on the corresponding number of students. The program was run to generate a report of the routes with the associated statistics as well as a graphic display of the routes.

**Sweep Method**
The sweep method is the simplest method of the three and allows for hand calculations. The sweep method is likely to violate time windows, however it allows for the generation of routing patterns in an expedient manner. It is a three-step process. The first step locates the depot and the stops on a map or grid. The second step draws a straight line from the depot to the arbitrarily chosen stop. The line sweeps clockwise over the stops adding them to the route. This process continues until the capacity is reached. The set of nodes visited thus far defines a route. A new stop is selected and the process continues until all of the stops have been assigned to routes. The last step entails sequencing the stops on the routes to minimize distance (7).

CASE STUDY

The study area will be Riverdale, in Morris County, located in Northern New Jersey. A map of Riverdale is shown as Figure 1. The borough has one combined elementary and junior high school with 261 students for grades K to 8. For the 1999-2000 school year, 199 students were bussed to school. Riverdale has a student mobility rate -- students entering and leaving the school during the school year -- over the last five years ranging from 4.4 to 14.6 percent. The mobility rate levels will be applied to the bus routing numbers, thereby estimating students requiring bus service to be between 184 to 214 students during the school year.

Several methods are used to determine which students are bussed. Students who live in the area East of Route 287, North of Route 23, and West of the Railroad tracks are bussed. Students who do not fit into these criteria are bussed if the walk is over 2.0 miles for grades K-6, and 2.5 miles for grades 7-8. Figure 1 shows a map determining of the bussing area. The 199 students who are currently being bussed are evenly distributed by weighted average of residencies. Next, the students are clustered into groups for the bus stops. The assignments are made with two conditions. First, bus stops must be 0.25 miles or less from the residences, and there can be a maximum of ten students per bus stop. Figure 1 shows the bus stop clusters with corresponding number of students. Three bus sizes are used: a 54-seat bus, a 20-seat bus and a 16-seat bus. They have operating costs of $60, $50, and $45 per hour, respectively.

RESULTS

Time Saving Heuristic

Since the service area covers both arterials and highway, two distance matrices are necessary: one for the arterials, and one for the highway. The average speeds of 15mi/hr for the arterials and 50mi/hr for the highway are used to calculate the time matrix. A time saving matrix generated using the Borough of Riverdale Zoning Map (8) is shown in Table 1.

As a primer of the method, a derivation of Route 1 is dicussed in detail. From Table 1 the largest savings is $s_{24,23}$. This means that stops 24 and 23 need to be connected. The maximum time saving related to node 24 is $s_{24,22} = 11.2$ minutes, while the maximim time saving related to node 23 is $s_{23,22} = 11.3$ minutes. Upon verification that neither the capacity nor the time window constraints are violated, node 22 is added to node 23. Node 23 grows and the route becomes 24-23-22. Now, there are 24 students on the bus. The growth nodes are now nodes 24 and 22. The maximum time saving related to node 24 is 7.6 minutes, the maximim time saving related to node 22 is 7.8 minutes. The capacity and time window constraints are not violated, thus node 22 grows and the new route becomes 24-23-22-21. The growth nodes are now
nodes 24 and 21. The maximum time saving related to node 24 is 6.1 minutes, the maximum time saving related to node 21 is 6.5 minutes. With the capacity and time window constraints not exceeded, node 20 is added, and the route becomes 24-23-22-21-20. The growth nodes are nodes 24 and 20. The maximum time saving related to node 24 is 5.4 minutes, the maximum time saving related to node 21 is 6.3 minutes. Since by adding any node the time window constraint would be violated, no new node is added. The first route is 0-24-23-22-21-20-0. The row and the column corresponding to nodes 20 and 24 are deleted from further consideration. The total operating cost for route 1 is $27.40.

Completing the process resulted in the following four routes: bus 2 (54 seats) on route 0-12-19-18-17-16-0 with an operating cost of $24.85; bus 3 (54 seats) on route 0-13-15-14-11-10-9-0 with an operating cost of $27.20; bus 4 (54 seats) on route 0-1-2-3-4-6-7-0 with an operating cost of $25.20; and bus 5 (20 seats) on route 0-8-5-0 with an operating cost of $8.25. A map of the school bus routes for the time saving heuristic methodology is found in Figure 2.

**ROUTER Method**

ROUTER required input on the depot and the stop coordinates, dwell times, students per stop, number and capacity of buses, operating cost per bus type, time window, and the speeds. After all the data is input, ROUTER performs a data check to validate the data, before generating the bus stops routes and route sequence.

ROUTER generates routes in both map and text format. For each bus route, the detail includes the starting time, dwell time, distance to next top, and if the time window is met. The utilization capacity, which is students/capacity, is calculated per stop. Cost information generated for all the routes includes the vehicle costs which includes mileage to the hundredth of mile and the operating costs which includes fixed costs as well as driver costs. Route 1, 0-20-19-21-22-23-24-0, has an operating time (including travel time and dwell time) of 29 minutes and the operating cost of $29.59. The remaining routes with the stop sequence and their costs are shown in Table 3. A map of the school bus routes derived from ROUTER is found in Figure 3.

**Sweep Method**

Due to the time window constraints, the sweep method was performed two times. The first time the sweep method was applied, it resulted in four routes with the lowest cost but one of the routes violated the time window constraint. The sweeping method was rerun; dividing the route that violated the time window constraint into two routes. The results are shown in Table 4. A map of the school bus routes can be found in Figure 4.

**CONCLUSION**

The results of the three methodologies used to optimize bus routes are compared, with ROUTER resulting in the lowest cost of $109.50. The results obtained from the time saving heuristic and the ROUTER method were very close and better than that obtained from the sweep method. All three methods optimized the problem with 5 bus routes; both the heuristic and ROUTER used four 54-seat buses and one 20-seat bus while the sweep method used four 54-seat buses and one 16-seat bus. Some of the differences are based on differing methods the three techniques used for calculating costs. The second
best method is the time savings heuristic with $112.90. The operating costs resulted in a $3.40 difference between ROUTER and the heuristic, and a $4.15 difference between ROUTER and the sweep method. As illustrated in Figure 2 and Figure 3, bus routes 3, 4 and 5 are identical in the heuristic and ROUTER, however the operating costs results are $1.21 different. The main difference between the heuristic and ROUTER lies in bus routes 1 and 2, where ROUTER has one additional stop for bus route 1 resulting in a $1.71 savings. While the sweep method generated different routes, all of the solutions between the three methodologies were within a range of less than 4 percent difference for the operating costs.

While the results were similar the performance measures of the three methods must be accounted for in the analysis. The traditional heuristic method required the most amount of time and the sweep method required the least. However, the most accurate of the methods is the heuristic method since it accounts for all of the various constraints as well as multiple depots. The ROUTER method allows for expeditious adjustments for changes the student mobility rate during the year. Multiple depots would make the ROUTER and sweep methods inefficient. The ROUTER program would require reprogramming and sweep methodology would require numerous iterations.

Evaluation of the three techniques can provide direction to school bus districts and bus operators to develop cost efficient and timely school bus routes and schedules. For districts that have one school or one depot, the ROUTER methods provides the best results. Once the data is input into the computer, slight adjustments in the number of students can easily provide new routes. With districts having several schools or depots, the time saving heuristic provides the best results.

The varying results and the performance measures indicate that the school bus routing problem will continue to be a source for analysis. There are several areas for future research for the problem reviewed in this paper. One area for future research includes developing time windows for the return trip and performing an overall optimization. Secondly, the variable demand can be considered to obtain optimal routes. Thirdly, the project could be re-analyzed using various bus capacities. It is recommended that sensitivity analysis be performed by varying the operating cost of the buses and the bus size, to observe the changes in routing. Another area for further research involves utilizing the same information in a Geographic Information System format to provide another means in evaluating the solution (9). The GIS application allows for multiple routing methods for larger school districts.

ACKNOWLEDGMENTS

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TABLE 1 Time Saving Matrix (Minutes)

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**TABLE 2 Optimization Results for Heuristic Methodology**

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<th>Number of Students</th>
<th>Vehicle Capacity</th>
<th>Capacity Utilization</th>
<th>Operating Costs</th>
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<td>54</td>
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Total:   

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TABLE 3 Optimization Results for ROUTER Methodology

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**TABLE 4 Optimization Results for the Sweep Methodology**

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FIGURE 1. Riverdale Map with Bus Stops
FIGURE 2. Heuristic Solution: Riverdale School Bus Routing
FIGURE 3. ROUTER Solution: Riverdale School Bus Routing
FIGURE 4. Sweep Solution: Riverdale School Bus Routing