TITLE: Evaluation of Feeder Bus Systems with Probabilistic Time - Varying Demands and Non-additive Time Costs

by

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EVALUATION OF FEEDER BUS SYSTEMS WITH PROBABILISTIC TIME-VARYING DEMANDS AND NONADDITIVE VALUE OF TIME

Steven Chien¹, Lazar N. Spasovic², and Renu S. Chhonkar³

ABSTRACT

The paper presents a comparison of optimal fixed route conventional bus (CBS) and flexible route subscription (SBS) bus systems. The systems are compared in terms of average cost per trip, including the operator and user costs, as an objective function to be minimized, with vehicle size, route spacing, zone size and headway as the system decision variables. The systems serve probabilistic demand that varies over a ten-hour operating period with a higher number of trips in the morning and afternoon periods. Passengers are assumed to have non-additive value of time. Average cost per trip is calculated for a numerical example in order to compare the suitability of a particular service under various demand conditions. For this particular example, the CBS has the lower cost service. However, the operator can further reduce the cost of daily operation by providing the CBS in periods of high demand and operating the SBS in off-peak periods. In general, the threshold value of demand at which one system is more cost effective than another is readily calculated. A sensitivity analysis is conducted to show the effect of varying model parameters on the objective functions and the decision variables.

KEYWORDS: Public Transit, Bus Service Planning, Paratransit.

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1. INTRODUCTION

Conventional Bus Systems (CBS) and paratransit systems, such as Subscription Bus Systems (SBS) differ in their service attributes and this is caused by the manner in which they operate. CBS are characterized by their fixed routes and schedules are generally economically viable to operate in a region with substantial demand density. SBS have flexible routes and/or schedules and are considered most suitable for the regions with low demand density. Circumstances under which it would be preferable to operate one system over the other should be determined in order to design an efficient public transportation system for a given service area.

The objective of this paper is to provide guidelines to determine when one system should be used over the other. The approach is to develop models of the two systems and to compare the total cost for operating an optimal CBS and an optimal SBS system that connects the service region and the Central Business District (CBD). The models’ objective functions minimize the total system cost (the sum of operator and user costs) for the CBS and the SBS, by optimizing the vehicle size and dimension of the service zone area. In addition, this study considers realistic issues in planning transit systems, such as probabilistic demand and the non-additive value of time. The non-additive value of time means that passengers place higher value on one 10-minute time segment (e.g., waiting for a bus) than on the ten 1-minute time segments.

The paper uses the average cost per trip as the criterion for determining which system is preferable and how it should be optimized under various circumstances. Solutions for both the optimal vehicle size and service zone area size are obtained for the CBS and the SBS. A sensitivity analysis is conducted to show the effect of varying model parameters on the objective functions and the decision variables. This is critical in deciding the suitability of a particular system given the prevailing conditions in the service area.

2. LITERATURE REVIEW

Scientists and engineers have studied extensively the vehicle routing problem for public transportation systems. Generally, these studies optimized system design variables while minimizing the cost functions subject to different constraints. For example, Gerrard (1) developed a probabilistic model of conventional taxi service (i.e., vehicle travel time was determined by a probability function) and applied it to a dial-a-ride system. Stein (2) solved a bus routing problem as the travelling salesman problem. He developed mathematical expressions for length and time of completion of tours for single and multiple bus systems that are asymptotically optimal. Lalena (3) developed a method of solving the travelling salesman problem using genetic algorithms. This method was able to quickly find near-optimal solutions for a large number of visited points.

The transit route planning problems are also discussed extensively in the public transit literature. Chang and Schonfeld (4) developed mathematical models for minimizing the total
system cost and used these models to compare a fixed route conventional bus and a flexible route subscription bus system and to determine the suitability of each system for different deterministic demand densities. Additive value of time was assumed. Gerrard et al. (5) optimized the service area by minimizing cost functions, while the time varying demand was considered. Bramel et al. (6) found an asymptotic optimal solution while routing vehicles in capacitated networks. Bramel and Simchi-Levi (7) applied heuristic methods to solve the vehicle routing problem in which customers requested service with certain probability during a time period. Braca et al. (8) developed a computerized heuristic method for routing and scheduling school buses operating in New York City. Chien and Schonfeld (9) minimized the total cost of an integrated bus and rail system using an iterative method. The optimal bus route spacing, headway and rail line length were optimized. Gabriel and Bernstein (10) solved the shortest path problem with non-additive value of time (i.e., the value of time function was nonlinear).

This paper formulates mathematical models for the comparison of the Conventional and Subscription Bus services. The paper is based on Chang and Schonfeld (4), but in marked contrast to that paper it has a probabilistic demand and a non-additive value of time.

3. MODEL FORMULATION

In order to develop models for both the CBS and the SBS, a rectangular service area ($L \times W$) that is $J$ miles away from the transportation terminal is assumed. The service area is divided into $N$ equal branch zones as shown in Figures 1 and 2.

For both the CBS and the SBS, buses travel from the terminal to the southwest corner of the service area with express speed. Then, for the CBS, the buses travel with local non-stop speed to the corner of the assigned zone, and with local collection speed in the assigned zone to pick up/deliver passengers. For the SBS, the buses travel with local non-stop speed to the corner of the assigned zone, and with a local collection speed within the assigned zone to pickup/deliver passengers.

Table 1 shows the definition of all the variables that are used in the model formulations.

Assumptions

1. The mean and variance of demand density (trips/mi$^2$/hour) is fixed and uniformly distributed over the service area and time period. It can vary over different time periods.
2. The bus headway in the SBS is equal for all zones in the service area and the operating cost is uniform within each period. The buses are filled when they complete the collection service in each zone.
3. The average wait time is equal to half the headway. The vehicle layover time is negligible.
4. Passengers perceive the time cost as non-additive. This means that for an average passenger the cost of one 10-minute wait is higher than the total combined cost of ten
1-minute waits. This is modeled by a non-linear average cost function: $\rho t^2$, where parameter $\rho$ takes into account different values that passengers place on in-vehicle time $v$, access time $x$, and wait time $w$, respectively.

5. SBS provides door to door service, thus the passenger access cost is negligible.

**Conventional Bus Systems (CBS)**

The total CBS system cost includes the operator cost $C_o$ and the user cost $C_u$. The hourly operator cost $C_o$ is the product of fleet size $F$ and vehicle operating cost $B$ (in dollars per vehicle hour)

$$C_o = FB$$

(1)

The fleet size $F$ is equal to the total vehicle round trip time divided by the headway $h$ and multiplied by the number of routes (W/r). It is given by:

$$F = \frac{WD}{Vrh}$$

(2)

where $D$ is the equivalent average bus round trip distance, $V$ is the local service speed, and $r$ is the route spacing. The equivalent average bus round trip distance is:

$$D = 2L + W / z + 2J / y$$

(3)

where $L$ is the length, and $W$ is the width of the service area, $z$ is the non-stop ratio (i.e., the ratio of local non-stop speed and local speed), $J$ is the line haul distance, and $y$ is the ratio of express speed and local speed. The round trip time is given as $D/V$.

The bus operating cost $B$ can be formulated as:

$$B = a + bS$$

(4)

where $a$ is the fixed cost coefficient, $b$ is the variable cost coefficient, and $S$ is the vehicle size.

The service headway $h$ is derived as

$$h = S / (rLQ)$$

(5)

where $Q$ is the average demand density.

Therefore, the hourly operator cost $C_o$ can be derived by substituting Eqs. 4 and 5 to Eq. 1 yielding:

$$C_o = \frac{LWQ.D(a + bS)}{VS}$$

(6)
The hourly user cost $C_u$ consists of in-vehicle cost $C_v$, wait cost $C_w$, and access cost $C_x$

$$C_u = C_v + C_w + C_x$$

The in-vehicle cost $C_v$ is equal to the demand $(LWQ)$ multiplied by the in-vehicle cost. The in-vehicle cost is equal to the total passenger trips between the service area and the terminal, $t_v$, multiplied by the value of in-vehicle cost per passenger $v(t_v)$. $v(t_v)$ is a non-linear function of average passenger in-vehicle time $t_v$, which can be formulated as:

$$t_v = \frac{L}{2V} + \frac{W}{2zV} + \frac{J}{yV} = \frac{M}{V} \quad (7)$$

where $M$ is average trip distance. Based on assumption 4, $C_v$ is

$$C_v = LWQv_t^2 = LWQv \left( \frac{M}{V} \right)^2 \quad (8)$$

Based on assumption 3, the average wait time is half the headway ($t_w = h/2$). Thus the hourly user wait cost $C_w$ can be similarly derived as

$$C_w = LWQw \left( \frac{h}{2} \right)^2 = LWQw \left( \frac{S}{2rLQ} \right)^2 \quad (9)$$

The average access distance to the nearest route is $r/4$ since the spacing between the adjacent branches of local bus service is $r$ and the service trip origins are uniformly distributed over the area. Similarly, if the stop spacing = $d$, the access distance alongside the route to the nearest transit stop is one fourth of the stop spacing $d/4$. Thus, the hourly access cost $C_x$ is equal to the demand $(LWQ)$ multiplied by the access cost $v(t_x)$. Thus,

$$C_x = LWQx \left[ \frac{(r + d)}{4g} \right]^2 \quad (10)$$

where $g$ is average walking speed. Therefore, the total system cost $C$ can be derived as:

$$C = C_v + C_u = \left[ D \left( \frac{a + bS}{VS} \right) + v \left( \frac{M}{V} \right)^2 + \left( \frac{S}{2rLQ} \right)^2 \frac{h^2}{2} + x \left( \frac{r + d}{4g} \right)^2 \right] LWQ \quad (11)$$
Since the demand is probabilistic and assumed to follow the Normal distribution, $Q$ in Eq. 11 can be formulated as:

$$Q = q(Q', \sigma)$$  \hspace{1cm} (12)

where $Q'$ and $\sigma$ are the mean and the standard deviation of demand, respectively.

The objective function to be minimized is the average cost per trip $c$ and can be formulated as:

$$\min c = \frac{D(a + hS)}{VS} + \sqrt{\frac{M}{V}} + w\left(\frac{S}{2rLq(Q', \sigma)}\right)^2 + x\left(\frac{r + d}{4g}\right)^2$$  \hspace{1cm} (13)

The decision variables in the objective function to be optimized are route spacing $r$ and vehicle size $S$.

**Subscription Bus Systems (SBS)**

In the SBS model, the collection tour distance $D_c$ in an optimized zone can be approximated by Stein (2) as given by:

$$D_c = k\sqrt{nA}$$  \hspace{1cm} (14)

where $n$ is the number of passengers in a collection tour. It is based on the assumption that $N$ points are randomly and independently dispersed over area $A$, and that an optimal traveling tour has been designed to cover these $N$ points.

The number $n$ is obtained by multiplying the trip demand in each zone $AQ$ by the headway $h$ and dividing it by the average number of passengers $u$ per pick-up point:

$$n = \frac{AQ}{u}$$  \hspace{1cm} (15)

Introducing Eq (15) into (14) yields:

$$D_c = kA\sqrt{Q/uh}$$  \hspace{1cm} (16)

The average line haul distance from the terminal to the center of a service zone is: $(L+W)/2 + J$. Therefore, the average vehicle round trip time $T$ is:

$$T = 2\left(\frac{L+W}{2zV} + \frac{J}{yV}\right) + \frac{D_c}{V} = \frac{D_c + kA\sqrt{Q/uh}}{V}$$  \hspace{1cm} (17)
where $D_L$ is the equivalent round trip distance from the terminal to the service zone. Similarly, the fleet size $F$ can be derived as:

$$F = \frac{LWT}{Ah} = \frac{LW(D_L + kA\sqrt{Qh/u})}{V Ah}$$

Thus, the operator cost $C_o$ is:

$$C_o = FB = \frac{LWB(D_L + kA\sqrt{Qh/u})}{V Ah}$$

where $B$ is the bus operating cost as shown in Eq. 4 for the CBS. Based on assumption 2, the headway $h$ is given by:

$$h = \frac{S}{AQ}$$

The total user cost, $C_u$, is the sum of user in-vehicle cost $C_v$, and user wait cost $C_w$

$$C_u = C_v + C_w$$

These costs are given by Eqs. (22) and (23) respectively:

$$C_v = LWQv\left[\frac{T}{2}\right]^2 = vLWQ\left[\frac{D_l + KA\sqrt{Qh/u}}{2V}\right]^2$$

$$C_w = LWQw\left[\frac{h}{2}\right]^2$$

Therefore, the total system cost $C$ is formulated as:

$$C = \frac{LWQD_l(a+bs)}{VS} + \frac{LWQk\sqrt{A/(uS)(a+bs)}}{V} + vLWQ\left(\frac{D_l}{2V}\right)^2 +$$

$$+ vLWQ\left(\frac{K\sqrt{A_S/u}}{2V}\right)^2 + wLWQ\left[\frac{S}{2AQ}\right]^2$$

The average cost per trip, $c$, is the total cost, $C$, divided by the total trip demand $LWQ$. The demand is given in Eq. (12). The objective function is to minimize the total average cost per trip.
The decision variables in the SBS objective function are the service zone area $A$ and the vehicle size $S$.

4. AN EXAMPLE

This section presents the optimal solutions that minimize the average cost per trip for the CBS and SBS. The input data are shown in Table 1. Stochastic demand was considered within the service area. The mean value of demand density ranges from 4 to 20 trips/km²/hr while the standard deviation varies from 0.8 to 2.5 trips/km²/hr during a 10-hour study period. The mean and the standard deviation vary over time and are shown in Table 2. The optimization software LINGO was used to optimize the decision variables by minimizing the objective function. The CBS decision variables were vehicle size and route spacing. The SBS model optimized vehicle size and zone size.

In this particular numerical example, the total daily cost of CBS service is $15,871. The cost of serving the same area using the SBS yields the total daily cost of $16,126, making the CBS the lower cost system. If the operator selects the least cost bus service for each period, the minimum total daily cost of operation is reduced to $15,803. Under this situation, the operator has to alter the type of service provided from period to period according to the following schedule: operate the CBS in periods one, three and five, and operate the SBS in periods two and four.

The comparison of the resulting optimal CBS and SBS system designs on the basis of average system cost, given in Figure 3, shows that the cost is equalized at the demand density level of 5.5 trips/km²/hr. If the demand density is lower than 5.5 trip/km²/hr, then the SBS is the lower cost system. Figure 4 shows the average user and operator costs ($/trip) for the two systems as a function of demand density. Tables 3 and 4 present the optimal values of the average system costs for the CBS and the SBS with respect to different demand densities and confidence intervals over different time periods.

5. SENSITIVITY ANALYSIS

A sensitivity analysis is performed by varying the values of parameters. Figure 5 presents the optimal headway for demand levels varying from 0.7 trips/km²/hr to 30 trips/km²/hr. For the CBS, the optimal headway ranges from 0.41 hr to 0.22 hr, while for the SBS, the optimal headway varies from 0.52 hr to 0.21 hr.
Figure 6 illustrates the optimal vehicle size for different demand densities. As the demand increases, the optimal vehicle size for the CBS increases faster than that of the SBS. Consequently, Figure 7 shows that as the demand increases the SBS fleet size increases faster than that of the CBS.

Figure 8 shows the variation in the total cost for the CBS and the SBS with different optimal vehicle sizes for different demand densities. As the demand density increases, both systems show a reduction in the cost with an increase in vehicle size.

In Figure 9, the variation of the total cost per hour for different confidence intervals (90%, 95% and 99%) reveals that the total cost of the SBS is more sensitive to the variation in the demand. Figure 10 illustrates the variation of the user and operator costs for different confidence intervals. The costs significantly increase as the confidence interval is increased.

Figure 11 presents the variation of the load factor for different confidence intervals. In all cases, the load factor decreases while the confidence interval increases because of the increased vehicle size. Also, the ratio of the standard deviation over mean, $\frac{\sigma}{Q'}$, increases as the load factor decreases. Figure 12 shows the histogram comparing the daily user and operator costs ($/10$-hr day) for the two systems. The figure shows that the SBS operator cost is higher than that of the CBS, while the CBS user cost is much higher as compared to that of the SBS.

6. CONCLUSIONS

The paper develops two models for optimizing conventional and subscription bus service system designs while considering probabilistic passenger demand density and non-additive value of time. Both models optimize the vehicle size. The CBS model optimizes route spacing, while the SBS model optimizes zone size. The optimal headways for both systems are derived based on the optimal values of decision variables. It has been demonstrated that the models can determine which system is most cost effective given particular characteristics of the demand function. Furthermore, the models provide the threshold level of demand density at which one system is preferable to the other.

In future research, some of the assumptions can be relaxed to take into account the vehicle layover time, and to allow the load factors to be greater than one. The models can also consider elastic demand (i.e., travel volume is sensitive to changes in service variable such as headway). This will enable the introduction of additional operator and user objective functions (e.g., maximization of social welfare) and lead toward more comprehensive policy analyses.
ACKNOWLEDGEMENTS

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REFERENCES


### TABLE 1. Variable Definitions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Fixed cost coefficient</td>
<td>$/hr</td>
<td>45.0</td>
</tr>
<tr>
<td>A</td>
<td>Service zone area (= LW/N')</td>
<td>km²</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>Variable cost coefficient</td>
<td>$/hr</td>
<td>0.3</td>
</tr>
<tr>
<td>B</td>
<td>Bus operating cost for period (i = a + bS)</td>
<td>$/veh-hr</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>Average total cost</td>
<td>$/trip</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Total cost</td>
<td>$/hr</td>
<td></td>
</tr>
<tr>
<td>(C_o)</td>
<td>Total operator cost</td>
<td>$/hr</td>
<td></td>
</tr>
<tr>
<td>(C_u)</td>
<td>Total user cost</td>
<td>$/hr</td>
<td></td>
</tr>
<tr>
<td>(C_v)</td>
<td>Total in-vehicle cost</td>
<td>$/hr</td>
<td></td>
</tr>
<tr>
<td>(C_w)</td>
<td>Total wait cost</td>
<td>$/hr</td>
<td></td>
</tr>
<tr>
<td>(C_x)</td>
<td>Total access cost</td>
<td>$/hr</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Bus stop spacing</td>
<td>km</td>
<td>0.3</td>
</tr>
<tr>
<td>D</td>
<td>Equiv. avg. bus round trip distance (= 2L+W/z+2J/y)</td>
<td>km</td>
<td></td>
</tr>
<tr>
<td>(D_k)</td>
<td>Distance of one collection tour</td>
<td>km</td>
<td></td>
</tr>
<tr>
<td>(D_t)</td>
<td>Equivalent line haul distance (= (L+W)/z+2J/y)</td>
<td>km</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>Load factor at the peak load point</td>
<td>pass/seat</td>
<td>1.0</td>
</tr>
<tr>
<td>F</td>
<td>Fleet size</td>
<td>veh</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>Access speed</td>
<td>km/hr</td>
<td>2.5</td>
</tr>
<tr>
<td>h</td>
<td>Headway</td>
<td>hr/veh</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Period index</td>
<td>(1–5)</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Number of periods</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>Line haul distance</td>
<td>km</td>
<td>8.0</td>
</tr>
<tr>
<td>k</td>
<td>Constant in the collection distance equation</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Length of service area</td>
<td>km</td>
<td>4.0</td>
</tr>
<tr>
<td>M</td>
<td>Equivalent avg. trip distance (= (L+W/z+2J/y)/2)</td>
<td>km</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>Number of passengers in one collection tour</td>
<td>pass</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Number of branched zones in conventional bus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N')</td>
<td>Number of service zones in subscription bus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Q_i)</td>
<td>Demand density for period (i)</td>
<td>trips/km²/hr</td>
<td>(15–40)</td>
</tr>
<tr>
<td>Q</td>
<td>Average demand density</td>
<td>trips/km²/hr</td>
<td>(25.0)</td>
</tr>
<tr>
<td>r</td>
<td>Route spacing (= L/N)</td>
<td>km</td>
<td></td>
</tr>
<tr>
<td>(R_{ij})</td>
<td>Ratio of std deviation over mean for each period (= \sigma_i/Q_i)</td>
<td>seats/veh</td>
<td>(.08–.24)</td>
</tr>
<tr>
<td>S</td>
<td>Vehicle size</td>
<td>seats/veh</td>
<td></td>
</tr>
<tr>
<td>(\sigma_i)</td>
<td>Standard deviation of demand density for period (i)</td>
<td>trips/km²/veh</td>
<td>(2.8–4.2)</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>hr</td>
<td></td>
</tr>
<tr>
<td>(t_v)</td>
<td>Average in-vehicle time for conventional bus</td>
<td>hr</td>
<td></td>
</tr>
<tr>
<td>(t_j)</td>
<td>Duration of service period (i)</td>
<td>hr</td>
<td>(1–3)</td>
</tr>
<tr>
<td>u</td>
<td>Average number of passenger per pickup point</td>
<td>pass</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>Local service speed</td>
<td>km/hr</td>
<td>18</td>
</tr>
<tr>
<td>v</td>
<td>Parameter of average in-vehicle time cost</td>
<td>$/pass-hr²</td>
<td>16.0</td>
</tr>
<tr>
<td>w</td>
<td>Parameter of average wait time cost</td>
<td>$/pass-hr²</td>
<td>36.0</td>
</tr>
<tr>
<td>W</td>
<td>Width of service area</td>
<td>km</td>
<td>3.0</td>
</tr>
<tr>
<td>x</td>
<td>Parameter of average access time cost</td>
<td>$/pass-hr²</td>
<td>36.0</td>
</tr>
<tr>
<td>y</td>
<td>Express speed/local speed ratio (convent. bus, subscrip. bus)</td>
<td></td>
<td>(1.8, 2.0)</td>
</tr>
<tr>
<td>z</td>
<td>Non-stop ratio (=) local non-stop speed/local speed</td>
<td></td>
<td>(1.8, 2.0)</td>
</tr>
</tbody>
</table>

Conventional bus \(z=1.8\), subscription bus \(z=2.0\)
### TABLE 2. Variation in Demand density over Time

<table>
<thead>
<tr>
<th>Time of the day</th>
<th>Duration (hr)</th>
<th>Demand Q (trips/km²/hr)</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>08:00 – 10:00</td>
<td>2</td>
<td>20</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>10:00 – 12:00</td>
<td>2</td>
<td>4</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>12:00 – 13:00</td>
<td>1</td>
<td>10</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>13:00 – 16:00</td>
<td>3</td>
<td>4</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>16:00 – 18:00</td>
<td>2</td>
<td>15</td>
<td>2.5</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3. Total Cost vs Demand for various Confidence Intervals for CBS

<table>
<thead>
<tr>
<th>Demand (trips/sq.km/hr)</th>
<th>Mean value</th>
<th>Total Cost ($/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CI: 90%</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>1.28</td>
</tr>
<tr>
<td>Q₁</td>
<td>20</td>
<td>2,797</td>
</tr>
<tr>
<td>Q₂</td>
<td>4</td>
<td>837</td>
</tr>
<tr>
<td>Q₃</td>
<td>10</td>
<td>1,632</td>
</tr>
<tr>
<td>Q₄</td>
<td>4</td>
<td>837</td>
</tr>
<tr>
<td>Q₅</td>
<td>15</td>
<td>2,229</td>
</tr>
</tbody>
</table>

### TABLE 4. Total Cost vs Demand for various Confidence Intervals for SBS

<table>
<thead>
<tr>
<th>Demand (trips/sq.km/hr)</th>
<th>Mean value</th>
<th>Total Cost ($/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CI: 90%</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>1.28</td>
</tr>
<tr>
<td>Q₁</td>
<td>20</td>
<td>2,879</td>
</tr>
<tr>
<td>Q₂</td>
<td>4</td>
<td>824</td>
</tr>
<tr>
<td>Q₃</td>
<td>10</td>
<td>1,667</td>
</tr>
<tr>
<td>Q₄</td>
<td>4</td>
<td>824</td>
</tr>
<tr>
<td>Q₅</td>
<td>15</td>
<td>2,291</td>
</tr>
</tbody>
</table>
Figure 1 Conventional Bus System (CBS)

**LEGEND:**

- Boundary between zones
- Bus with express speed $yV$
- Bus with non stop local speed $zV$
- Bus with local speed $V$
FIGURE 2  Subscription Bus System (SBS)

LEGEND:

--- Bus with express speed $yV$

---- Bus with non stop local speed $zV$

--- Bus with local collection speed $V$

Collection Area

J = Line haul distance

W

L

Transportation Terminal
FIGURE 3 Optimal average cost for the CBS and SBS

Average Cost, $/trip

Demand, Q (trips/sq.km/hr)

TC CBS
TC SBS

FIGURE 4 Optimal average User and Operator Costs for CBS and SBS

Cost ($/trip)

Demand, Q (trips/sq.km/hr)

UC CBS
UC SBS
OC SBS
OC CBS
FIGURE 5  Headway vs Demand

FIGURE 6  Vehicle Size vs Demand
FIGURE 7  Fleet Size vs Demand

FIGURE 8  Total Cost Vs Optimal Vehicle Size
FIGURE 11 Load Factor vs Confidence Interval for various time periods

FIGURE 12 Daily Operator and User Costs for the CBS and SBS